ARTICLE V.

Memoir explanatory of a New Perpetual Calendar, Civil and Ecclesiastical, Julian and Gregorian. By William McIlvaine. Read August 15, 1845.

When my attention at school was first turned to the solution of chronological problems, I received a strong impression that the Dominical Letters formed too complex and cumbrous an appendage of the calendar. According to the directions given in Pike's Arithmetic, at that time a standard elementary work, one of the first seven letters of the alphabet, in regular order, A always standing at the first day of January, must be prefixed to every day in the common year of 365 days. The letter which in this series happened to be affixed to the first Sunday in January was called the Dominical Letter of that year, as marking all its Sundays, but in leap years the next following letter was to serve as the Dominical Letter for the first two months of the year. A, B, C, D, E, F, G, with 1, 2, 3, 4, 5, 6, 7, subjoined, constituted a table, the numbers being an index to the letters. By a rule, whose principle was left unexplained, the sum of the Julian year, its fourth part *and four* was to be divided by *seven*. If no remainder occurred, G was the Dominical Letter, but if any number remained *that* number of letters, beginning the reckoning with F, was to be counted in a retrograde order from G, to reach the Dominical Letter; or on deducting the remainder from *seven* you might obtain the index of the Dominical Letter, counting forwards from A. The rule was shorter for Gregorian years, but in both styles the Sunday letter was double in every leap year, and after the proper letter had been found it was required to count backwards again in order to know on what day of the week the first of January happened. Finally, if the day of the week corresponding with a given day of any other month was wanted, the tedious process of adding the day of the month to the number of days in all the preceding months, and dividing the sum by *seven*, must be gone through, and the answer deduced from another remainder.

The whole system seemed to me "cycle and epicycle, orb in orb," the week appearing to revolve about the letter A, and the year around the week. I had imagined the structure of the Julian year, when once divested of the absurdity of calends, nones, and ides, (all counted backwards,) and after having the week introduced, to be essentially simple; and although the Dominical Letters, for ages consecrated to clerical use, had, I knew, been adopted by the ablest astronomers and chronologists of modern times, and were still employed conjointly with the golden number, by learned judges of the King's Bench, in England, for ascertaining the legal periods of the four terms of court, yet I could not help believing that some less circuitous method would sooner or later supersede an apparatus which, however true in its results, and however lucid in comparison with some other parts of the machinery, causes most persons, from its complexity alone, to consider even the *civil calendar* a subject of much difficulty, and *that of the Church* as an impenetrable mystery. I have, accordingly, always looked with particular interest at the *Perpetual* Almanacs, so called, (but serving only for a single century,) which have since fallen in my way, with a vague hope of being able to discover in them the germ of some simpler mode of computation.

On consulting, about two years ago, the *seventh* edition of the Encyclopedia Britannica, I met accidentally with a new, well-digested, and perspicuous article on the "Calendar," from the pen of Thomas Galloway, Esq., F. R. S., one of the Vice-presidents of the Royal Astronomical Society, and was gratified at learning from it how much had been done within the present half-century, by the analytical skill of Gauss and Delambre, towards disencumbering the calendar of numerous tables, and substituting for them plainer formulæ. I perceived, however, and not without disappointment, that the Dominical Letters were still retained among the elements of the algebraical equations, and on applying them arithmetically I found my computation embarrassed at the outset by the quantity $7m$ joined with many quantities differing in sign, contained in the first equation, and at the close, by another inconvenient equation, $p = P + (L - l,)$ involving the value not only of the Sunday letter, but of a letter belonging to the fifteenth day of the calendar moon.

After a careful reperusal of Mr. Galloway's essay, I was struck with the fact to which I had never before adverted, that the first year of the Christian era began with Saturday, which, being the seventh day of the week, and corresponding with *seven*, the number of the hebdomadal cycle, suggested to me the possibility of making that number a convenient starting-point of an indefinitely prolonged succession of ages or chronological periods, in which the ordinal numbers appropriate to the other six days of the week, might perform in a direct and more natural and intelligible manner, the same function that the Dominical Letters, in a reversed order and in connexion with the Solar Cycle, had heretofore done. Pursuing, as the amusement of many leisure hours, this casual thought, I have succeeded in constructing what I believe to be a *new instrument*, of plain materials, but rather better adapted than the old, for ordinary, popular use, and perhaps of a labour-saving character, even in the hands of the learned; although to this last consideration I wish to be understood as not attaching much importance, but merely as asking the opinion of mathematicians in regard to the plan, if, at first blush, they should deem it at all worthy of

their scrutiny. To them a difference of "methods," technically so called, being, for the most part, of little consequence, my proposal to "*free* the civil calendar from algebraic formulæ," (which are apt to convey, to their minds, the best evidence and guidance,) may at first sound strangely; but the attempt will possibly appear to them, upon reflection, quite consonant with the improved spirit of the present age, which brings, whenever practicable, things of common concern and use to common comprehension. In regard to the Church calendar, indeed, which Delambre declares to be "excessivement compliqué et qui, pour être bien compris, exige l'attention la plus soutenue," I must admit it to be doubtful whether in any shape yet assumed, it is likely to become an inviting or fruitful object of inquiry to the generality of persons.

The next page exhibits, and the succeeding one exemplifies my plan, the constituent parts of which are embodied in a single Tablet or Perpetual Calendar, having its peculiarities specified in the title. In addition to the two general *Rules*, it will be seen to consist, principally, of a central column, headed "Eras," which serves as an index to two series of corrections, called (in compliance with customary fictions and nomenclature,) the Solar and Lunisolar Equations. These are comprised in two side columns of equal length with the central one, and accompanying it; column A being appropriated to civil, and column C to ecclesiastical purposes.

The Julian Era, though employed by astronomers as a convenient universal measure of time, is here limited to years *after* Christ, but embraces, of course, all those years which *precede* the adoption of the reformed calendar, at various epochs, by the several nations of Christendom. That is to say, the equations 5 and 0 respectively, standing at the head of the side columns, and "beside" the Era denominated "Julian after Christ," are intended to be used *constantly* until such a reformation has actually taken place. The Era denominated "Gregorian from 1582," is divided into centurial figures, but the equations standing beside these subdivisions are not meant to be applied to the interval, for example, of nearly 170 years which occurred between the adoption of the New Style at Rome, and in Great Britain. Without this preliminary caution, the word "Eras," as employed in the tablet, might possibly mislead some readers.

The asterisks annexed to the centurial figures are not essential to the use of the Tablet, but are retained for the purpose of better elucidating its structure. The intervals between them on the *civil* side, make known, however, at once, through the eye, those *centesimal* years which, in New Style, are not bissextile; and they furnish also a ready means of ascertaining the whole number of days, by which, at any period after 1600, the two styles differ from each other. For instance, in years having the centurial figures 26, the difference between Old and New Style amounts to 18 days: for 10 (the number of days originally suppressed,) together with the 8 centurial figures which are destitute of asterisks, (viz., 17, 18, 19, 21, 22, 23, 25 and 26,) make the sum 18: in like manner, the difference, with our present centurial figures is 12 days: for $10 + 2$ (17 and 18 being unmarked) $= 12$.

The two auxiliary tables B and D demand no attention until some progress shall have been made in the explanations of the general rules, upon which I am now to enter.

PERPETUAL CALENDAR,

CIVIL AND ECCLESIASTICAL,

Freed from Dominical Letters, Solar Cycle, Golden Numbers, Extended Tables of Epacts and Algebraic Formulæ. By Wm. McIlvaine. 1844.

RULE.

To find on what day of the week a given day of the month falls in any year after CHRIST, to the end of Time.

Add to the Year,
its **4**th part, (omitting fractions,)
the Eq. in column **A.** beside the Era,
the *number for the month* in Table **B**,
and the Day of the Month.
(or the excess over 7s in the Day of the Month.)

Divide the sum by 7,

The excess over **7**s, calling **0** always **7**, will be the day sought, viz.:

The	**1**st,	**2**d,	**3**d,	**4**th,	**5**th,	**6**th,	**7**th,
Being	**Su.**	**M.**	**Tu.**	**W.**	**Th.**	**F.**	**Sa.**

Except in Jan. and Feb. of Leap years, when the *preceding* will be the true day.

All JULIAN Years, when multiples of **4**, are Leap Years. GREGORIAN years, when multiples of **4**, are also Leap Years, *Unless* their two right hand figures be **00**, joined with left hand or centurial figures which are *not* multiples of **4**.

Table B.

Numbers for the Months.

		Feb.		
Ist Quarter. Jan.	**0**	**3**	**3**	Mar.
IId. " April	**6**	May. **1**	**4**	June
IIId. " July	**6**	Aug. **2**	**5**	Sept.
IVth. " Oct.	**0**	**3**	**5**	Dec.
		Nov.		

The No. for *each* Month in Table **B.** is the number of days beyond full weeks, in *all* the Months which precede it from the beginning of a Common Year.

The order and position of the figures **3**, **6**, **5**, (the number of days in a common Year,) and of **1**, **2**, **3**, **4**, **5**, in the space between them, may recall to memory the whole Table.

Solar Equation.	ERAS.	Lunisolar Equation.
A	**Julian,** After Christ, UNVARIED.	**C**
5		0
	Gregorian, From 1582, VARIED BY Centurial figs.	
2	**15**	1
2	* **16**	1
1	**17**	0
0	**18** *	0
6	**19**	29
6	* **20**	29
5	**21** *	29
4	**22**	28
3	**23**	27
3	* **24** *	28
2	'**25**	27
1	**26**	26
0	**27** *	26
0	* **28**	26
6	**29**	25
5	**30** *	25
4	**31**	24
4	* **32**	24
3	**33** *	24
2	**34**	23
1	**35**	22
1	* **36** *	23
0	**37**	22
6	**38**	21
5	**39** *	21
5	* **40**	21
4	**41**	20
3	**42**	19
2	**43** *	19
2	* **44**	19
1	**45**	18
0	**46** *	18
6	**47**	17
6	* **48**	17
5	**49** *	17
4	**50**	16
3	**51**	15
3	* **52** *	16
2	**53**	15
1	**54**	14
0	**55** *	14
0	* **56**	14
6	**57**	13
5	**58** *	13
4	**59**	12
4	* **60**	12
3	**61** *	12
2	**62**	11
1	**63**	10
1	* **64** *	11
0	**65**	10
6	**66**	9
5	**67**	8
5	* **68** *	9
4	**69**	8
3	**70**	7
2	**71** *	7
2	* **72**	7

RULE.

To find the Calendar-Moon's age on the 1st day of January in any Year, from the beginning of the CHRISTIAN ERA.

Add to ten times the Year,
the Year,
† its **19**th part, (omitting fractions,)
and the Eq. in col. **C**, beside the Era.

† (But if no fraction, add 1 less than the 19th part *in Julian Years.*)

Divide the sum by 30,

The Excess over **30**s. calling **0** = **30**, will be the Age sought, or *Annual Epact.*

Table D.

TO FIND EASTER SUBTRACT ANY EPACT				The Diff. will be the DAY of the MONTH on which occurs the Paschal Term.
GREGORIAN.		**JULIAN.**		
Between	From	Between	From	
1 & 12	**13**	1 & 4	**5**	in April.
13 & 23	**44**	5 & 15	**36**	in March.
‡25 & 30	**43**	17 & 28	**35**	in April.
Subtract 24	From **42**	Subtract 29	From **34**	

‡ 25 also from 42, if in getting a 19th of the year the Remainder exceed 10.

' This is the Epact 25' of the Tables in the Article "Calendar," Encyclopedia Britannica, 7th Edition.

Then find by the CIVIL CALENDAR the Day of the Week on which *that* Day of the Month falls, and the following Sunday will be EASTER SUNDAY.

The number of days to be counted forwards will be shown by subtracting *the Day of the Week* thus found, from 8.

EXAMPLES.

OLD STYLE.

What Day of the Week was April 2d, A. D. *326?*

4)	326
	81
A	5
Mo.	6
Day	2
7)	420
	60
Remainder	0
or	7
Answer Saturday, Thence	

Required Easter, A. D. 326.

	3260
19)	326
	17
C	0
30)	3603
	120
Rem. or Epact	3
Taken from	5
Term *April,*	2
. . . *to Sunday*	1
Answer April	3

NEW STYLE.

What Day of the Week will be March 22d, A.D. 1845?

4)	1845
	461
A	0
Mo.	3
Day	22
7)	2331
	333
Remainder	0
or	7
Answer Saturday, Thence	

Required Easter, A.D. 1845.

	18450
19)	1845
	97
C	0
30)	20392
	679
Epact	22
From	44
Term *March*	22
. . . *to Sunday*	1
Answer March	23

Rule proved by examples from De Morgan. See British Almanac and Companion, for 1845.

Julian Year.

4)	1639	
	409	
A	5	
Mo.	6	
Day	10	
7)	2069	Rem.
Wednesday,	4	
From	8	
	4	

Easter.

	16390
19)	1639
	86
C	0
30)	18115
Epact	25
From	35
Term April	10
4 . . . . . to Sunday	4
Same Answer, April	14

Gregorian Year.

4)	4610	
	1152	
A	0	
Mo.	6	
Day	13	
7)	5781	Rem.
Friday	6	
From	8	
	2	

Easter.

	46100
19)	4610
	242
C	18
30)	50970
Epact	30
From	43
Term April	13
2 . . . . . . to Sunday	2
Same Answer, April	15

Rule proved by examples from Delambre. See Conn. des Tems for 1817, *and Hist. de l'Astron. Mod.*

Julian Year.

4)	4763	
	1190	
A	5	
Mo.	6	
Day	12	
7)	5976	Rem.
Thursday	5	
From	8	
	3	

Easter.

	47630
19)	4763
	250
C	0
30)	52643
Epact	23
From	35
Term April	12
3 . . . . . to Sunday,	3
Same Answer, April	15

Gregorian Year.

4)	3909	
	977	
A	5	
Mo.	6	
Day	17	
7)	4914	Rem.
Saturday	0 = 7	
From	8	
	1	

Easter.

	39090
19)	3909
	205
C	21
30)	43225
Epact	25'
From	42
Term April	17
1 . . . . . . to Sunday	1
Same Answer, April	18

The point of view in which the subject presented itself will probably be best understood by expanding my original course of reasoning into figures in the following way:

The 1st day of year 1 of Christ, having the Sunday letter B, was SATURDAY, or the 7th day of the week; which number 7 agreeing with that of the weekly cycle, (never to be interrupted,) suggests this plan of freeing the calendar from Dominical Letters.

If (referring to the following Table, SERIES I.,) to the 1st day of the year 1 we add 5, in order to reach and include the 7th or last day of that cycle, the sum 7 divided by 7, gives us the remainder 0; which remainder being always taken as the equivalent of 7, the divisor (conformably to arithmetical usage in the case of all other cycles,) becomes a fit expression for SATURDAY: and 0, or a week completed, will thus represent perpetually that day of the week in the scale of time.

The same process with the succeeding ordinal days of the year, exhibits a perfectly correct expression for the other six intermediate days of the week, as in

SERIES I.—*Days of the Common Year.*

													NOTE.
To day the . .	1st	2d	3d	4th	5th	6th	7th	8th	15th	22d	29th	365th	Rejecting 7s from 365 leaves 1
Of year . . .	1	1	1	1	1	1	1	1	1	1	1	1	Then adding year 1
Add constantly, .	5	5	5	5	5	5	5	5	5	5	5	5	And Constant 5
Divide by 7 the sum	7	8	9	10	11	12	13	14	21	28	35	371	We get the sum 7
There remains .	0	1	2	3	4	5	6	0	0	0	0	0	And same Rem'r 0
Or	SAT.	SU.	M.	T.	W.	TH.	F.	SAT.	SAT.	SAT.	SAT.	SAT.	or SAT.

Showing that

Year 1 *begins* and *ends* (52 entire weeks, or 364 days intervening) on 0 or SATURDAY.

A like Table for succeeding years would show

in year 2, its *first* and *last* day, . . . by the Remainder 1 to be SUNDAY,
in year 3, its *first* and *last* day, . . . by the Remainder 2 to be MONDAY.

Year 4 is a leap year, but not until the 29th of February;
its *first* day, therefore, would be shown . . by the Remainder 3 to be TUESDAY,
and its *last*, or 365th day, ... by the Remainder 4 to be WEDNESDAY.

This result for the end of the year 4 would be the same if, instead of calling its last the 366th day, . . we add to the 365th day of a common year,
the year 4
then, by the general rule, its fourth part 1 for the first Leap Year of the Era,
and constant 5

The sum . . . . . . 375 divided by 7, gives Remainder 4 or WEDNESDAY for the 31st of December. But this rule, as applied to the beginning or 1st day of January of the year 4, would make it also to be Wednesday, which is not the true day, but *Tuesday is*, as we have seen above. Hence the exception stated in regard to the months of January and February in leap years, viz., to take the *preceding* as the true day, since no intercalation occurs until the 29th of February. I proceed to show the effect of *seven* such *quadrennial* intercalations to advance by exactly a week, the day of the week, after going through the first complete cycle of the Julian Era.

Accordingly, using as before the day of the year and the constant, which for our present purpose (see NOTE to *day* 365th,) are together, always 6, or (1 + 5) the

Year 5,	with its $\frac{1}{4}$ part,	(a whole No.)	or 1,	added to	6,	make the sum	12	and Rem.	5.	It begins and ends on	TH.
6	"	"	or 1,	"	6	"	13	"	6	"	F.
7	"	"	or 1,	"	6	"	14	"	0	"	SAT.
8	"	2, less 1 till Mar.	or 1,	"	6	"	15	"	1	It begins	SU.
9	"	2, less 0 after Feb.	or 2,	"	6	"	16	"	2	It ends	M.
"	"		or 2,	"	6	"	17	"	3	It begins and ends on	TU.
10	"		or 2,	"	6	"	18	"	4	"	W.
11	"		or 2,	"	6	"	19	"	5	"	TH.
12	"	3, less 1 till Mar.	or 2,	"	6	"	20	"	6	It begins	F.
"	"	3, less 0 after Feb.	or 3,	"	6	"	21	"	0	It ends	SAT.
13	"		or 3	"	6	"	22	"	1	It begins and ends on	SU.
14	"		or 3	"	6	"	23	"	2	"	M.
15	"		or 3	"	6	"	24	"	3	"	T.
16	"	4, less 1 till Mar.	or 3	"	6	"	25	"	4	It begins	W.
"	"	4, less 0 after Feb.	or 4	"	6	"	26	"	5	It ends	TH.
17	"		or 4	"	6	"	27	"	6	It begins and ends on	F.
18	"		or 4	"	6	"	28	"	0	"	SAT.
19	"		or 4	"	6	"	29	"	1	"	SU.
20	"	5, less 1 till Mar.	or 4	"	6	"	30	"	2	It begins, ...	M.
"	"	5, less 0 after Feb.	or 5	"	6	"	31	"	3	It ends	T.
21	"		or 5	"	6	"	32	"	4	It begins and ends on	W.
22	"		or 5	"	6	"	33	"	5	"	TH.
23	"		or 5	"	6	"	34	"	6	"	F.
24	"	6, less 1 till Mar.	or 5	"	6	"	35	"	0	It begins	SAT.
"	"	6, less 0 after Feb.	or 6	"	6	"	36	"	1	It ends	SU.
25	"		or 6	"	6	"	37	"	2	It begins and ends on	M.
26	"		or 6	"	6	"	38	"	3	"	T.
27	"		or 6	"	6	"	39	"	4	"	W.
28	"	7, less 1 till Mar.	or 6	"	6	"	40	"	5	It begins	TH.
"	"	7, less 0 after Feb.	or 7	"	6	"	41	"	6	It ends	F.
29	"		or 7	"	6	"	42	"	0	It begins and ends on	SAT.

In this list of years, SATURDAY corresponds in one leap year (12,) with the *last* day of the year only; in another leap year (24,) with the *first* day only; and in two common years, (7 and 18,) being the *third* and *second* after leap year, with *both the first and last day:* but it is in years 1 and 29 alone, that all the conditions of the Julian Year become exactly alike.

Thus, after a lapse of 28 entire years,

(of which the .	21 common years contain .	7665 days, or exactly .	1095 weeks,
and the . . .	7 leap years contain . . .	2562 days, or exactly .	366 weeks,
making in all .	28 years, or	10227 days, or	1461 weeks,)

between the first days of January in the years 1 and 29, both of these years being the *first* after bissextile, SATURDAY returns to be the first and also the last day of the year. In the years 30, 31, 32, &c., down to 57, all the days of the week will recur in the same order as in 2, 3, 4, down to 29, at the beginning and end of years having a common relation with leap year, and so on *for ever.*

The rule of the Tablet is therefore perfectly consonant with the well-known law of the Solar Cycle and Dominical Letters in the Julian Calendar, and it is evident to the mere arithmetician, that the part which the Equation 5 plays, in the type or first cycle of that Calendar, just exhibited at large in connexion with the initial and final *day of the year*, would be equally well performed by it, in conjunction with any intermediate ordinal day of those years, or with any other Old Style year, without limitation.

But thus far we have not considered the year as divided into *months and days of the month*, both of which are embraced as elements of computation in our general rule.

On carrying out the plan of the foregoing Table, and introducing every month, it will be seen that the *first* days of each month are equivalent to the following ordinals of the common year, placed in the upper line of

SERIES II.—*First Days of the Months.*

Months . . .	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Day of the Year	1	32	60	91	121	152	182	213	244	274	305	335
Year, ...	1	1	1	1	1	1	1	1	1	1	1	1
Constant . . .	5	5	5	5	5	5	5	5	5	5	5	5
Sum divided by 7	7	38	66	97	127	158	188	210	250	280	311	341
Remainder . .	0	3	3	6	1	4	6	2	5	0	3	5
or Day of the W'k	SAT.	TU.	TU.	FR.	SU.	W.	FR.	M.	TH.	SAT.	TU.	TH.

Now these Twelve Remainders disposed in Quarters of a year, (and in a form and order quite as easily remembered as the usual tabular index to the Dominical Letters,) constitute Table B of the Perpetual Calendar. They are called Nos. for the respective months, and on being substituted for the corresponding ordinals of the year standing in the upper line, will, when the first day of the month comes to be added with them to the year and the constant 5, yield the very same remainders, and so indicate correctly the day of the week on which each month of the first year of the Christian Era begins.

0, the monthly number for January, marks the true zero of the civil year, or the midnight between the old and the new year: and this No. added to the day of the month, equals throughout January the ordinal day of the year. The rest of the monthly Nos. mark the midnights preceding the first day of each subsequent month, expressed in the odd days over full weeks, counted from the same zero-point: and this distance, together with the days of each month in succession, must, of course, equal the other ordinals throughout the year. But since the rejection of entire weeks (or division by 7,) does not affect the day of the week, the sum of the monthly No. and the day of the month, may be substituted uniformly for those ordinals. Thus the 31st of December, whose monthly No. is 5, added to the year 1, its fourth part 0, and the constant 5 = 42, which sum divided by 7 gives us 0, or Saturday, the same result as when calling it the 365th day in SERIES I. Either an additional year, or an additional day of the month, or an additional 29th of February in every fourth year, advances by 1 the day of the week as regularly and permanently, through an infinite succession of weekly cycles, as we have already found them to do, in the computations connected with that series, in which 0, the No. for January, with 1, the day of the month, might have been used as equivalent to 1, the day of the year.

It is demonstrated, therefore, that the use of Dominical Letters, whether single or double, may be entirely dispensed with in the *Julian* Calendar. The Solar Equation or Constant 5, standing at the head of column A, effects for ever, with the aid of Table B, by a process almost purely additive, the same object in a simpler and easier way; and supplies the place, not only of the whole apparatus of *Regulars* and *Concurrents* heretofore combined with the *Solar Cycle*, but also of the memorial lines usually employed for ascertaining the first day of every month, in a manner which was not, however, readily applicable when the year happened to begin on any other day than Sunday. It may not be unworthy of a passing notice that (regarding the solar regulars *one* and *eight* as the same,) each of the twelve monthly numbers of Table B, viz., 0, 3, 3, 6, 1, 4, 6, 2, 5, 0, 3, 5, is less by 2 than each of the ancient Solar Regulars, 2, 5, 5, 1, 3, 6, 1, 4, 7, 2, 5, 7, and is less by 1 than the respective numbers . . . 1, 4, 4, 7, 2, 5, 7, 3, 6, 1, 4, 6, representing the order, in the alphabet, of the initials A, D, D, G, B, E, G, C, F, A, D, F, of the twelve words composing the doggerel rhyme which we have just referred to, namely, "At Dover Dwell, George Brown Esquire; Góod Christopher Finch, And David Friar."

From the Constant 5, thus perfectly established as a starting point, is readily deduced the retreating series of small secular equations which follow it in the same column, and these will, in turn, answer equally well for finding the day of the week belonging to any day of any month, in any year throughout the *Gregorian* Era. · For the *first* step in the reformation of the Julian Calendar, in 1582, taken for the purpose of restoring the equinoxes to their former place in the year, consisted in the suppression of 10 days in that year by calling the day which was the fifth of October, in the Old Style, the 15th of October in the New. On applying the Rule given in the Tablet, the Julian 5th of October, 1582, will be shown by the Remainder 6, to have been *Friday*, but the Julian 15th of October, being a week and three days farther on, would, by the same rule, be found to happen on *Monday*. Now the series of days of the *week* was not interrupted, nor intended to be, by the reform. The days of the *month only* were to be differently named after 10 of them should be expunged. The epoch, day, or point of time called in the Julian Calendar, the 5th of October, must continue to be Friday in the Gregorian, and be referred to as the 6th day of the week, whatever new denomination it might receive as a day of the month. In order, then, to make the 15th of October in the New Style correspond with Friday, it is obvious that we must go back three days in the calculation, or in other words, that from the Julian Solar (or rather Hebdomadal) Equation of . . . 5
must be deducted, the excess over a full week of the 10 days lost, or 3
and that thenceforward, during a certain period, the Gregorian solar correction or —
equation must be 2
which number stands opposite, in Column A, to the first centurial figures, namely to 15 and 16.

This equation 2 would for ever perform the same office in the Gregorian that 5 had done, and still does, in the Julian Calendar, but for a *second* step taken at the Reformation. With a view to prevent in future that deviation of the nominal civil *Days* from the definite *Seasons* of the year which had arisen from introducing too many leap years, it was then determined that after 1600, which continued bissextile in both Calendars, every suc-

ceeding hundredth year whose centurial figures were not divisible by 4 without a remainder, should cease to be leap years, but that every 400th year whose centurial figures were multiples of 4, should continue to be leap years. These multiples, 16, 20, 24, &c., are, accordingly all marked on the Civil side of the Calendar with asterisks, but the intermediate centurial figures 17, 18, 19, and 21, 22, 23, &c., on the same side, are left unmarked.

Now whenever an asterisk occurs, no change takes place in the solar equation; but at each of the three other centurial figures that equation is diminished by a unit, on account of the one day lost at each successive non-intercalation of the 29th of February. The equation 2 *beside* 15, continues, of course, to be 2 at 16. From 16 downwards, the equations limited by the weekly cycle of 7, (which number is always represented by 0 in column A,) fall into sets of *four* each, in a receding series, each new set of four beginning with the same figure with which the last set ended; and 2 followed by 2 reappears at the centurial figures 43, 44, &c., and at 71, 72, &c., thus returning after *four times seven*, or twenty-eight centuries, to the same figure, or Solar Equation 2, and so on *ad infinitum*.

Column A, consisting of fewer figures (and these symmetrically disposed in a cycle of 7) than have ever been used in constructing any Table of Dominical Letters for *either* style, completes, accordingly, A CIVIL CALENDAR of simple form and unlimited range.

From the terms of the Rule, it is obvious that the Remainder on division by 7 of the first three items, (viz., the given year, its fourth part, and the secular correction in column A,) forms a standing number, which, being once obtained and noted on New Year's day, may serve a convenient purpose throughout that year. This Remainder is universally the complement to 8 of the Dominical number for the year, and might be called the *Yearly Number*.

Then, supposing the Monthly Numbers well fixed in the memory, (a task which the division of Table B into a thick-lined polygon, resembling a carpenter's square, containing twice 3, 6, 0; and a thick-lined square containing a 5, leaving an interval composed of squares less strongly marked, but numbered in regular order, 1, 2, 3, 4, 5, greatly facilitates,) the day of the week will be readily found without resort to an Almanac, by adding together the Yearly No., the Monthly No., and the Day of the Month, and rejecting the sevens from their sum. Since this sum never exceeds 43, the whole process may be *mentally* performed without difficulty, after a little practice. During the present century, whose solar equation is 0, the computation is particularly easy; for instance,

The Remainder or Yearly No. for 1845, to be kept in mind is $\left(\frac{1845 + 461 + 0}{7} = \right) 3$

What day of the week, then, is August the 15th? Add that day of the month, . . 15

And the Monthly Number, 2

The sum is 20

which, divided by 7, (or mentally rejecting the sevens,) leaves the Remainder . . 6

or Friday. In leap years the exceptions respecting January and February must, of course, be attended to. Those exceptions might, instead of referring to the *preceding* day of the week, have been equally well provided for by the following direction, viz., "In January and February of leap Years, use the monthly numbers of July and August, in each case six months distant."

The solar equation belonging to any Gregorian century beyond the limits of the Tablet might be found, by numbering from 0 to 28, the equations in Column A, opposite to the centurial figures lying between 28 and 56. For, since the secular corrections recur in like order, at every succeeding period of four times seven centuries, *that* equation to which the Remainder, on dividing the given centurial figures by 28, stood attached, would be the Equation required, which may likewise be obtained without the use of tables, by a short rule given hereafter, near the close of the memoir.

I proceed to explain, as briefly as possible, the construction of the Ecclesiastical side of the Calendar, and the means I adopted, soon after the civil side had been completed, of connecting them with each other, thus making two of the monthly numbers in Table B, namely 3 and 6, still representing March and April, contribute towards shortening the calculations respecting Easter; and causing also a single additional Column, C, to serve as a convenient substitute for an extended table of thirty lines of Epacts, indexed with as many alphabetical letters, great and small, and consisting of nineteen numbers in each line.

Aided by Mr. Galloway's article, before referred to, and by one of Lord Macclesfield's, published in the Philosophical Transactions of 1750, (No. 494, page 417,) I found the task less difficult than I had anticipated. They describe the ingenious, but involved and intricate mode of expressing, in the Church Calendar, successive differences between Solar and Lunar years, by means of Epacts or Increments. These Epacts are so derived from the Golden Numbers, (that is, from the order in which the years stand in the Metonic cycle of 19 years,) as to indicate the age, at the beginning of each year, of an imaginary moon, whose artificial phases, though approaching, seldom correspond with, but are generally a day or two in advance of, the mean movements of the true moon. They state that, supposing the Epact of the year 1 to be 11, (that is, the difference between the common Solar year of 365 days and the Lunar year of 354 days,) the Epact of each following year of the *first* cycle of 19 is obtained, by adding 11 to that of the former year, and by rejecting 30, as often as the sum exceeds 30; but that at the 20th, 39th, 58th, 77th, &c., years, viz., at the beginnings of each succeeding cycle, 12 is to be added to the epact of the last year of each preceding cycle, and continued augmentations of 11, and rejections of 30, are to take place as before. Hence I inferred that the Golden Numbers, as *Remainders* on division by 19 of the year *plus* 1, might be dispensed with, and their place in computation be conveniently supplied, by adding to 11 times the year, the 19th part of the year, used as a quotient, or *whole Number;* taking care only that, when the Julian year happens to be a multiple of 19, one less than the 19th part shall be added. This easy formula yielded me, without a failure, the constantly recurring 19 Epacts that mark the ancient calendar, beginning with 11 and ending their round with 29; and the General Rule at the head of my Tablet, as there modified, is precisely an equivalent for it, but provides, at the same time, a practical advantage in the arrangement of the figures. The omission of fractions advances by 1, in the order prescribed, each successive set or cycle of Epacts, while the exception stated, guards effectually against the intercalation of 1 taking place at any earlier date than the proper cycle; just as, in the Civil Calendar, 1 less than one-4th part of the year is added in January and February of leap years. The reason

is the same also, for omitting the fractions occurring between multiples of 19, in the Church Calendar, as those occurring between multiples of 4, in the Civil.

With a view to demonstrate, without a large array of figures, the consistency of my Rule with well-established tabular modes of finding the Epact, I refer to the extended Table of Epacts in the Encyclopedia Britannica, (article "Calendar," page 12,) where, in line *c*, beginning with 11, will be found all the Julian Epacts, under a Golden Number, however, always one behind, or one less than, that which was originally assigned to the year. A. D. 1 was, in the old Calendar, always regarded as year 2, of the cycle of 19. This relation between the Golden Numbers and the Epacts was changed at the reformation, when the line D, beginning with 1, was selected for the Gregorian Epacts between the years 1582 and 1699, inclusive, and the . . . . Epacts 1, 12, 23, 4, &c.
were made to correspond with the Golden Numbers . . . . 1, 2, 3, 4, &c.

The Julian Epact for 1582, found by my Rule, and confirmed by the elaborate chronological Table contained in "The Art of Verifying Dates," a work of great learning and acknowledged authority, is 25, the same that is presented in the line *c*, of the extended Table of Epacts, under the Golden Number 5, (that of the Julian year being 6.) Descending *ten places* in that *column*, on account of the ten days suppressed at the reform, or, what comes to the same thing, deducting from the Julian Epact of 25
the number of days suppressed, or 10

we reach, or obtain . 15
But the advance of one Golden Number in the *line* D is equivalent to the addition of 11

and thus causes the Gregorian Epact for 1582 to be 26;
for 1583, to be 7; for 1584 to be 18; and so on; that is to say, makes it always greater by 1 than the Julian, until 1700, unless the years be multiples of 19, in which case the Julian Epact, as the Exception provides, will become always 29, and the Gregorian will be 1. The 15, in the transition from one style to the other, is not an Epact of the year in either, but a connecting link between the two. The Gregorian Epact, at that epoch, is in reality greater by a unit than the Julian; and the Julian Equation, standing at the head of Column C in my Tablet, being 0 *for ever*, the proper starting-point for the Gregorian Equations is 1, which number accordingly stands at the right hand of the centurial figures 15 and 16.

The succeeding Lunisolar equations, 0, 29, 28, &c., were reached in the following manner. Lord Macclesfield's directions for determining "in what years the Epacts should either be extraordinarily augmented or diminished, and the Golden Numbers should either be set backwards or forwards in the Calendar," according to the divisibility of the even hundreds by 3 or by 4, separately, or by both 3 and 4, or by neither, led me to the expedient, similar to the one I had already adopted in regard to the secular equations in the Civil Calendar, of marking with an asterisk, in strict obedience to the Gregorian law, every centurial figure at which, in successive periods of 25 centuries, (beginning at 1800, 4300, 6800, &c.,) the Epact is to be increased by a *unit*. This correction occurs at the end of every 300 years, 7 times in succession, and then *once* at the end of 400 years, making 8 corrections in the course of 2500 years.

The mere relative position of these right-hand asterisks, to those already placed on the left, serves to ascertain without calculation, the coincidence or otherwise, of the prescribed secular adjustments; and to indicate at once, by a process rather more direct and simple than Lord Macclesfield's, the joint effect upon the Calendar Moon's Age, of the *omission*, three times in every 400 years, of a 29th of February, on the one hand; and of the *addition*, eight times in every 2500 years, of an *extra*-Epact, on the other. For it is obvious, with reference to the normal bissextile intercalation of the Julian Calendar, and to its recurring Epacts, which depend upon the Golden Nos., and are, through them, connected with the Gregorian Epacts, by a definite law both of analogy and deviation, that on the *left*, an asterisk means *deduct nothing;* a blank means *deduct one day:* and that on the *right*, an asterisk means *add one day;* a blank means *add nothing*.

NOW having at the Cent. Figs. 15 and 16 as a fixed point of departure, the Equation 1†

if	at	17	we deduct	1	and add	0	it falls to	0 or 30
if	at	18	"	1	"	1	it remains	0
if	at	19	"	1	"	0	it falls to	29
if	at	20	"	0	"	0	it remains	29
if	at	21	"	1	"	1	it remains	29
if	at	22	"	1	"	0	it falls to	28
if	at	23	"	1	"	0	it falls to	27
if	at	24	"	0	"	1	it rises to	28
if	at	25	"	1	"	0	falls again to	27
if	at	26	"	1	"	0	it falls to	26, &c.

WHENCE we draw the following *general*, and almost mechanical, *Rule* for obtaining the whole series of Lunisolar Equations in Column C. viz.:

Descending from century to century in the central Column of Gregorian Eras,
Keep the Equation the same as before, when a single * occurs on *either* side;
Diminish the last equation by 1 when the *asterisk* appears on *neither* side;
And *increase* the last equation by 1 when the * * appear on *both* sides;
but limiting, of course, this generally, but not uniformly, receding series by the cycle of 30 or 0. In this manner is readily produced a succession of suitable Epacts, susceptible of indefinite extension, and requiring no supplementary precepts, such as Lord Macclesfield conceived would become necessary after the year 4199.

Before that distant age arrives, the Civil Calendar will doubtless undergo the slight modification generally recommended by modern astronomers, of omitting a single bissextile intercalation at every 4000th year, which would maintain an almost perfect accordance between the seasons and the beginnings of the year, for a thousand centuries to come; and the Ecclesiastical Calendar also will most probably receive, cotemporaneously,

† Although the Equation 1, on the right of the Centurial figures 16, results necessarily from the arbitrary appropriation of the Epact 26, with the Golden Number 6, to the first year of the Reformed Calendar, and does not depend upon, (but is the same with,) the preceding Equation, it nevertheless *happens*, in consequence of no leap year being suppressed until 1700, and of no Epact being added until 1800, to obey the general law of the asterisks, as expressed in the text.

some new adjustment. But, pursuing, without regard to future reforms of either Calendar, the plan I have just described as far as the 87th centurial figures, and comparing, at every step, the *results* derived from my General Rule, when applying the successive equations so obtained, with the *Epacts* set down in the extended Tables, I found them to be in exact correspondence with each other, line by line, and letter by letter, throughout the circuit, from little *a* to capital *P*. In fact, the equations in Column C are identical with the figures, which stand in the first column of Table II. (Encyclop. Brit., Art. "Calendar,") under the Golden Number 1, and immediately by the side of the 30 letters running up the column from D to C. Delambre pronounces the Epact-letters useless; I trust, therefore, that for discarding them, in company with the Dominical, from my Tablet, I shall not incur the reproach of being hostile to *Letters in general.*

The Gregorian Annual Epact being thus accessible, with very little more trouble than the ordinary process of finding the Golden Number, (since three, out of the four lines of figures to be added together, require no computation, but merely to be set down on paper in the order stated,) it remains for me to elucidate my mode of deducing from the Epact the Paschal Term, or the 14th day of the Paschal Moon, (most commonly, but improperly called "The Paschal Full-Moon,") on which Easter Sunday depends.

The Paschal Moon is that whose 14th day, counting the new moon as the *first*, never falls earlier than the 21st of March, nor later than the 18th of April, reckoning the calendar lunations of those months respectively, to contain 30 and 29 days; for Easter, according to the usage of the western churches of the Roman empire, sanctioned by the Council of Nice, must be celebrated on a Sunday, which Sunday must *follow* the 14th day of the Paschal Moon, so that whenever the Paschal 14th, or *Term* fell on Sunday, Easter could not arrive until the *next Sunday.* If the Term fell on Saturday, Easter came *one* day after that Saturday, viz., on the *next day;* consequently the interval between the Paschal Term and Easter might be 7 days, but could never be less than one day: and since the *Term* could not happen before the Calendar-Vernal Equinox, (which, whatever might be the astronomical fact, was, by the Church, invariably fixed on the 21st of March,) Easter Sunday, it is clear, must have its place between the 22d of March and the 25th of April, both days inclusive.

Now supposing the 14th day of the Paschal Moon to coincide with the Calendar Equinox, that moon must have been new, or *in its first day* on the 8th of March, (for 21 *less* 13, is 8,) and the preceding moon must have been 23 days old on the 1st of March, just a week earlier, (for 8 *less* 7, equals 1.) That is to say, the moon on the 1st day of March, must have been of exactly the same age as it was on the 1st of January; the interval between those days in common years, being two lunations, (one of 30, the other of 29 days,) or 59 days in all. The Epacts belonging to January and March are, of course, identical, as they are seen to be in every *Epact Almanac:* and it is manifest that when the Calendar-Moon's age on the first day of the year, exceeds 23 days, (in which case the new moon of March would happen before the 8th, and its 14th day, consequently, before the Equinox,) the *Paschal New Moon* will be in April, and when 23 or less, will happen in March.

If, then, the Paschal Term occurs on the 21st of March, the Annual Epact is 23
if on the 22d " it is 22
if on the 23d " it is 21
and so on, up . . . to the 31st " and down to the Epact 13,
the day of the month rising by *one*, as regularly as the Epact descends by *one*, so that their sum is always the same, ... viz. 44,
and the Paschal Term in March must, of course, be that number *less* the Epact.

Advancing with the Day of the Month to the 1st of April, we descend to the Epact 12
to the 2d " to 11
to the 3d " to 10
and so on, up . . . . . to the 12th " and down to the Epact 1,
the sum being always 13,
and the Paschal Term in that part of April which precedes the 13th, is 13 *less* the Epact.

Arriving, in like manner, at the 13th of April, we have the Epact 0, or 30
at the 14th " 29
at the 15th " 28
and so on, up . . . . . to the 18th " and down to the Epact 25,
the sum being always 43,
and the Paschal Term for the rest of April is 43 (or 30 + 13) *less* the Epact; subject, however, to two exceptions in the case of the *double Epacts* (25′-26, and 25-24.)*

The Gregorian Calendar of Epacts (see Table III., Encyclop. Brit., Art. "Calendar,") has been so constructed, that the Epact 25 belongs, whenever the Golden Number exceeds 11, to the same day with the Epact 26, and is then marked with an accent, to distinguish it. Now since 43, *less* 26, gives the 17th day of April as the proper Paschal Term or limit; 25′ in order to produce the same result, must be subtracted from 42.

By a like contrivance, the Epact 25 (not accented) belongs, when the Golden Number does not exceed 11, to the same day with the Epact 24; and since 43, *less* 25, yields the 18th of April as the proper Paschal Term or Limit; 24, in order to produce the same result, must likewise be subtracted from . . . 42.

In this manner both the exceptions are readily provided for, with a single diminuend (42.)

* This artifice, employed in the construction of the Table of Gregorian Epacts, by making the age of the moon to differ, occasionally, a day more from the truth, than it would otherwise have done, preserves between the Old and New Calendars, a certain conventional resemblance, which consists in the non-repetition of a given annual epact within the same lunar cycle. For the reason of this arbitrary mode of writing the epacts, and of varying, consequently, the Paschal Term, I must refer to regular treatises on the subject, remarking only that this device of Clavius, to which I have adjusted the *operation* of my own, does by no means prevent, in either style, *Easter Sunday* from falling, repeatedly, in the course of any single cycle of 19 years, *twice*, and sometimes *thrice*, on the same day of the month. In the last lunar cycle, for instance, ending with 1843, the New Style Easter occurred *thrice* on the 3d day of April, and in duplicate four times, on other days of the month. In the present cycle, ending in 1862, like duplicate Easters, on certain days of the month, will happen six times. In the course of the very first cycle of the reform at Rome, similar coincidences took place three times in New Style, and would have done so seven times in Old Style, if the Julian mode of reckoning had not been abandoned.

The Golden Number, being the remainder after division by 19 of the year *plus* 1, the excess of 10, where *the year itself* is to be divided by 19, agreeably to my rule, becomes an equivalent expression for the excess of 11 in the tables referred to.

It will be perceived that the Gregorian Epacts in Table D of my Tablet, are merely arranged in sets or sequences in a more orderly manner, beginning at 1 and ending at 30, than they presented themselves in the course of the foregoing explanations, and that the short column of four numbers, from which they are, according to their several places in the series, to be deducted, namely 13, 44, 43, and 42, assumes, at the same time, greater symmetry.

The difference between those numbers and the Epact can never, in April, exceed 18, but whenever the 14th day of the Paschal Moon falls on the 18th day of that month, and happens at the same time to be Sunday, Easter Sunday must be a week later, or the 25th of April. It must always be at least one day after the Paschal Term, but whether it is to be celebrated *one* or *seven*, or any intermediate number of days after that *term*, will always be correctly determined, when the day of the week on which the term occurs has been found by the Civil Calendar. It is obvious that the number of days to be counted forwards, must be the difference between 8 and the number which indicates the day of the week.

Before the close of the past year, I had prepared, in manuscript, a specimen of the Perpetual Calendar described in the foregoing pages, and had solved, by means of it, numerous chronological questions, trying uniformly the correctness of my results by the tables and formulæ of Lord Macclesfield and Mr. Galloway, when I learned from the Journal of the Franklin Institute, for November, that the next British Almanac and Companion would contain an Article from the pen of Mr. Augustus De Morgan, of University College, London, on the subject of a controversy not unlikely to arise, respecting the proper day of celebrating Easter in 1845. My rules gave me, as the Tablet shows, the 23d of March; but being naturally desirous of testing their accuracy, in that as well as other instances, by such distinguished mathematical authority, I awaited the receipt of a copy of that instructive Annual, which a friend had ordered from England, and which he kindly placed in my hands in the course of the winter. As it embraced, what I had anticipated, a very learned, interesting, full, and satisfactory Essay "on the Ecclesiastical Calendar," I could not but be highly gratified at finding my rules confirmed by every example therein computed, and more especially by the discovery that, owing to the *mechanism* of my Tablet, which spared me the labour of several arithmetical steps, which Mr. De Morgan, (who proposed to himself, and has ably executed, the more difficult task of pure calculation according to Delambre's analytical formulæ, and independent of all tabulation,) had been compelled, under the assigned limits of his problem, to take, I was enabled to attain, in much less time, by rules occupying less space, and requiring less mental and manual effort, the same results with those yielded by his fifteen "short arithmetical directions," given at pages 15 and 16. It appeared to me, besides, that the risk of error was considerably lessened in my process, by the operation's consisting, for the most part, either of the mere *setting down*, or of the *addition* of figures, these figures being,

in two of the items, equations obtained at a single inspection, and, in three other items, *the year itself*. It is for others, however, to decide, (and I should most cheerfully submit the question to the two above-named eminent living authorities, on whom I have relied,) whether the Tablet, which I now present to the Society as a *new* "*method*," professing only to combine very *condensed tables* with *plain rules*, may not possess a sufficient degree of practical utility to deserve publication.

Intent upon rendering the Tablet useful in calculations relating principally to the New Style, I should not have thought of embracing in it the ancient Church Calendar, now gone into disuse in almost every part of the Christian world, except the empire of Russia, had I not met with the "very simple Table" given by Mr. De Morgan, at page 32, and which he says "will supply the place of all rules *as soon as the Golden Number and Dominical Letter are known*." Now, since it has been steadily my aim to eliminate both those portions of the scaffolding of the Calendar, my work seemed incomplete unless I could still dispense with them, and could devise some other means, at least as easy in practice as the ten rules placed by Mr. De Morgan on the subsequent page, for determining the Old Style Easter.

I sought in vain for a copy of Clavius in the public libraries of the United States,* but was fortunate enough to find, in the Library of this Society, "The Art of Verifying Dates," (L'Art de vérifier les dates, troisième édition, par un religieux Bénédictin de la congrégation de St. Maur, Tome premier, à Paris, 1783,) whose very extensive Tables assisted greatly the accomplishment of my object. The "Chronological Table," and the "Perpetual Lunar Calendar," both clearly exhibit the same constant concurrence between the Paschal 14th of the Moon, and the place of the year in the cycle of 19, which is mentioned by Mr. De Morgan, and shown in his Table. We have therefore only to translate into their corresponding Epacts, the several Golden Numbers, in the order in which they there stand, and we shall obtain a series of 19 Julian Epacts, descending while the days of the month rise, as we before witnessed in the case of the Gregorian Calendar; with this difference in the exposition, that blanks are left at each of the ten places between the 21st of March and the 18th of April, where there is no golden number, and where there can be no Epact, since the numbers wanting to complete a regular series (viz., 13, 10, 8, 5, 2, 30, 27, 24, 21 and 19, as well as 16,) had no existence as Epacts in the Ancient Calendar. The Epact 29, forms a case by itself, on account of the peculiar interval of 12 *between* it and the *succeeding* Epact 11. In the Reformed Calendar the interval of 12 always *precedes* the Epact, belonging to every year which is a multiple of 19. But if the Julian XXIX. be treated like the Gregorian 0, or XXX., and be made to change places with 30, it ceases to appear anomalous.

Compare the following columns with those at page 117 of this memoir. The leading Epact there was 23, and is here 15.

* There is one, I have since learned, in the Library of Harvard University, at Cambridge, Massachusetts.

Golden Numbers.	Paschal Term.	Epacts.	Sum of Paschal Term and Epact.
16	March 21	15	
5	22	14	
.	.	.	
13	24	12	
2	25	11	
.	.	.	36
10	27	9	
.	.	.	
18	29	7	
7	30	6	
.	.	.	
15	April 1	4	
4	2	3	5
.	.	.	
12	4	1	
1	5,	0 or 29	34
.	.	.	
9	7	28	
.	.	.	
17	9	26	
6	10	25	
.	.	.	
14	12	23	35
3	13	22	
.	.	.	
11	15	20	
.	.	.	
19	17	18	
8	18	17	

These Epacts, on being re-arranged, in Table D, as the Gregorian were, produce a like symmetry in the column of Numbers 5, 36, 35, 34, from which they are to be deducted, and these numbers differ throughout, by exactly 8, from those which occupy the Gregorian column. The Golden Numbers, too, beginning with 16 and ending with 8, advance regularly by 8, rejecting 19 whenever the addition of 8 to the preceding Golden Number would make the next exceed 19: or, what comes to the same thing, entering line *c* in Table II., (Encyclop. Brit., Art. "Calendar,") and beginning at Epact 15, proceed always from the left to the right, as in a circle, with intervals of 8 places, leaping backwards from the end to the beginning of the line as often as may be necessary, and the whole of the succeeding Julian Epacts, which I have just presented, down to 17, will come regularly into view. The unit's figure of each Epact, except 29, is *one* behind the unit's figure of each Golden number, thus exemplifying a passing observation made at page 114, line 5th.

These curious relations between the Reformed and Ancient Calendar would perhaps be more strongly impressed upon the memory, if exhibited together, in the following form:

Take any Gregorian annual Epact	*To obtain the Paschal Term*	Take any Julian annual Epact
below 13 from . . . 13,	*in April*	below 5 from 5,
between 13 & 23 from 13 + 31 = 44,	*in March*	between 5 & 15 from 5 + 31 = 36,
above 23 from 13 + 30 = 43;	*in April*	above 15 from 5 + 30 = 35;
But take 24 & 25' from 13 + 29 = 42.	*in April*	*But* take 29 from 5 + 29 = 34.

The memory might be technically aided by adverting to the fact, that 31, 30, and 29 days, are the *length* of Civil months in all Leap Years, as well as that of artificial Lunations in the existing Church Calendar; and that the 13th and 5th were the days of the month on which the Ides and Nones, in every month but four, of the old Roman Calendar began.

It is of more importance, however, to state, that, having procured, in April last, through the politeness of Professor E. Otis Kendall, from the Library of the High School of Philadelphia, the use of Delambre's History of Modern Astronomy, I have, since then, calculated, according to my Tablet, every example given in Book I., "on the reformation of the Calendar," (including the extreme cases put by Delambre, to illustrate his principles, in the controversy with Ciccolini, about the relative merits of their respective formulæ,) and that I have found my results not only to correspond uniformly with those which are there computed, but to be reached much more expeditiously also, than by the processes there employed.

I have, moreover, been prompted by curiosity, as well as by caution, to extend my secular corrections as far as Delambre had pursued his, in the Connoissance des Tems, for 1817, and to carry down, accordingly, with the mechanism of the asterisks, my equations in Columns A and C, to the 500th century. I compared, as I proceeded, a few cases, at various epochs, and many, near the termination of my task, with the answers contained in his beautiful "Table Pascale," at page 43, Book I. (Astron. Moderne,) where the Gregorian Easters are obtained by inspection after the Dominical Letter and Epact are known; and I had the pleasure to find, in every instance, the results still identical.

Without doubting for a moment, or presuming to call in question, the correctness of the principles on which that Paschal Table has been constructed, I feel it, nevertheless, incumbent on me to remark, that, from the text of Delambre, at page 24, it would appear that Clavius's Great Table, which is almost universally consulted as a standard, differs from Delambre's small one, by including, among the years in which Easter happens on the 22d of *March*, the year 3860. According to Delambre's formulæ and Paschal Table, (the Dominical Letter being G, and the Epact 24,) it will fall on the 22d of *April;* according to Gauss's formula, (M being 2 and N being 5,) on the same day; by the tables of Lord Macclesfield and Mr. Galloway, (the Golden Number being 4, and the Dominical Letter G,) on the same day; and, finally, by Mr. De Morgan's Rules, and by my own, on the same day, that is, on the 22d of April. With such a preponderance of concurrent testimony in favour of the conclusion at which I arrive, I cannot but suppose that Clavius

must either be in error, or that the *month* has been, by some inadvertence, misquoted from his Table by Delambre. The day (or quantième) of the month is the same, according to all the mathematicians thus far named, and the *month* must be April, for the 22d of March corresponds with Thursday, and not with Sunday, in that year, and the Paschal New Moon cannot happen in March when the Epact exceeds 23. It seems no less unaccountable, that Sir Harris Nicolas, who gratefully acknowledges some assistance from Mr. De Morgan, when preparing his "Chronology of History," published in 1833, in Lardner's Cabinet Cyclopedia, should make the Easter of 3860, by the NUMBER OF DIRECTION in his Table H, to be the 15th of April,—just a week earlier than my instructors have computed it, and yet not agreeing with Clavius's decision, as before reported. I shall be glad if the pointing out of this strange discrepancy should lead to the removal of a radical error existing *somewhere*, and, as I believe, in Vince's Complete System of Astronomy, vol. 1, page 581, in Ferguson's Astronomy, and in Rees's Cyclopedia, (Article "Cycle,") in all of which, as well as in many other scientific and popular works of reference, a like Table, containing numbers of *Direction*, (so called,) said to be "adapted to the New Style," is given, but in terms unsanctioned by those later and abler *Analysts* whom I have consulted, and on whose principles I have relied. Certain it is, that Sir H. Nicolas's Tables, H and K, (pages 57 and 58, Edition II.,) are entirely at variance with each other in regard to the Easter day of every year having the Golden Number 14, (or Epact 24,) between 1590 and 1685, both inclusive; and it is equally true that Delambre's formulæ do not authenticate the Easters of any year having the Golden Number 4, (or Epact 24,) from 3803 to 3898, both inclusive, as calculated by Sir H. Nicolas's Table H. In these last cases the deviation varies from one to two weeks; in the former, it amounts always to four weeks.

The learned Benedictine of St. Maur, who, between the first and third edition of "The Art of Verifying Dates," discovered the means of reducing his "Perpetual Solar Calendar" from 210 folio pages, which he had confessed to be *rather* voluminous, to its present bulk of 42, makes the following truly philosophical reflection: "Tel est le sort des inventions humaines, de n' être perfectionnées que par degrés, et presque jamais du premier coup." Then, after explaining by what expedients he had accomplished so important an abridgment, he congratulates himself and the public on his great success, and finishes, by saying: "Il est simple, il est court." On examining his work, it will be evident, that 64 of his pages are essentially replaced by the single one forming my Tablet, which may, consequently, with more truth be characterised as "simple and short," without being found, I trust, the less sure.*

* N. B.—The words "*in Julian Years,*" standing at the close of the Exception to the General Rule for finding the Epact, were inadvertently omitted in the Tablet, as printed in the Quarterly Proceedings of 1845. This *Erratum*, (for the exception was not meant to be applied to Gregorian Years,) grew out of a change made by me, with a view to brevity, but without due care, in the terms of the original manuscript, which directed "that every *zero* resulting from the division of the sum by 30 should be called XXX. in New Style, but XXIX. in Old,"—a restriction exactly equivalent to the one here prescribed, and *that* by which all the computations connected with my device were really governed.

The facility of this method arises principally from the Gregorian Secular Equations, in Columns A and C, being *mechanically* formed and tabulated. To obtain the same corrections, *arithmetically*, no simpler rules than the following could perhaps be devised; and a comparison of the two processes will therefore exhibit fairly the economy of time and trouble which the Tablet effects, apart from what is saved by using Table B.

Rule for finding, universally, the New Style Solar Equation of Column A.

Divide the Centurial figures by 8, and the Remainder by 4.
same
From the sum of the two Quotients (increased, if needful, by 7,) deduct the Remainder.
The Difference (rejecting the sevens, if any,) will be the Equation wanted.
(But since the final rejection of the sevens, is provided for by the Rule of the Tablet, the Difference itself will answer every purpose of that Equation.)

Rules for finding the New Style Lunisolar Equation of Column C, until A. D. 1899 *inclusive.*

Divide the Centurial figures by both 3 and 4, and to the two Quotients add 8.
From the Sum deduct the Centurial figures, the Difference will be the Equation wanted.

From A. D. 1899 *to* A. D. 4199, *both inclusive.*

Divide the Centurial figures by both 3 and 4, and to the two Quotients add 8.
From the Centurial figures deduct the Sum: Then take the Difference from the next higher multiple of 30 (which, during this period, is always 30,) and the Second Difference will be the Equation wanted.

At and after A. D. 4200 *for ever.*

The rule is the same as the last, with this Exception only, viz.
that the Centurial figures must be diminished by $\left(\frac{C-17}{25}\right)_w = \left(\frac{4\,C-68}{100}\right)_w$,
that is to say, by the Quotient arising from the division of "four times the Centurial figures" *less* 68, by 100, before being divided by 3: but they must be used, in the rest of the work, exactly in the manner there directed.

The following results of the Rule and Tablet will be found in perfect accordance with those derived from Delambre's formulæ and Paschal Table, in determining Easter for A. D. 50000.

The Solar Equation of Column A = 3; and Lunisolar Equation of Column C = 3.
The Dominical Letters are *B* and *A*, and the Epact is 4.
The 14th day of the Paschal Moon will fall on Sunday, the 9th of April, and Easter, of course, on the following Sunday, viz., the 16th of April.

A collateral proof of the correctness of these secular equations is derived from the fact, lately observed by me, that the same mechanical expedient of the asterisks furnishes, instantly, all the Gregorian Corrections M and N of Gauss's formula, beginning with 22 and 2, but in advancing series (the reverse of mine) as given by M. Le Chevalier de Grésy, in Vol. XXIV., page 77, of the Memoirs of the Royal Academy of Turin.

SUPPLEMENT TO MR. McILVAINE'S MEMOIR.

Read December 18th, 1846.

When constructing the "New Perpetual Calendar," which I had the honour of presenting to the Society, last year, I purposely avoided furnishing any rule for the Era before Christ, lest the requisite explanations in regard to leap years, (which, for that period, are so expressed by chronologists, as never to be multiples of 4,) might unduly extend or complicate the Tablet. Since its publication, however, in the Quarterly Bulletin, where it appears in a reduced form, with the Examples conveniently separated from the Rules, I have been led to believe that, without exceeding the limits of a single page, in the next volume of the Transactions, at large, the 1st of the annexed Supplemental Rules might readily be subjoined to it, and thus render the plan applicable to *all Time.*

The Rule is a mere corollary from the general principles of the Tablet; for after A.D. 1, the numbers of the two series 1, 29, 57, &c., and 1, 20, 39, &c., (see pages 109 and 118) never again *coincide* until A.D. 533, 1065, 1597, &c., each of which years, like A.D. 1, is the first after leap year, begins and ends on Sat., and has the same Epact, 11. Reversing the order of time, the chronol. year 1, B. C., or astron. year 0, is the 28th of the Solar, and 19th of the Lunar, cycle, next preceding A.D. 1. It is a leap year, begins on Th., ends on F., has the Epact 29, and corresponds with A.D. 532. The chronol. year 2, B. C., or the astron. year 1, is the 27th of the Solar, and 18th of the Lunar Cycle, next preceding A.D. 1. It is the third after leap year, begins and ends on W., has the Epact 18, and corresponds with A.D. 531; and so *backwards* without limit, through all the combinations of Old Style cycles concurring at regular intervals of 532 *entire* years.

I have added a convenient Rule, the 2d, for years of the Julian Period; a 3d Rule, with formulæ, to serve as proofs; also, a simple method, Rule 4th, of solving *converse* problems, which, though more curious, perhaps, than useful, I beg leave to append to the memoir.

I was recently much gratified by learning from Mr. Galloway and Mr. De Morgan, who have, at my request, had the goodness to consult for me the fifth volume of Clavius's mathematical works, and to furnish me with a few particulars from the explication there given of the Reformed Roman Calendar, that my conjecture respecting the discrepancy, in a single case, between my results and those *set down* in the great Table of Clavius, as well as in the text of Delambre, turns out to be correct. On tracing the difference to its source, it appears that the Table and the text are alike *erroneous*, but accidentally so.

The substance of the communications with which I have been honoured on the point in question, is this. In the Table there is an "*obvious misprint*," at the angle where the line belonging to the year 3860 meets the column of Easters; in which column the months of March and April are not otherwise distinguished than by their initial letters M and A. The Easter for that year "stands 22 M, but ought to be 22 A;" for, in the same line, and in nearly adjoining columns on the left, the Paschal full moon is made to fall "18^{d} 18^{h} A," the Paschal 14th is made "18 A," and all the moveable feasts *accord* with those of an Easter that occurs on the 22d day of April. Moreover, in the column immediately on the right stands "Pentecost 10 June," which is absurd, when Easter is any where in March, and renders the fact more strange that Delambre should have "*missed seeing the error.*"

SUPPLEMENTAL RULES

EXTENDING AND FACILITATING THE USE OF THE FOREGOING TABLET.

1*st.* *A Rule for Julian Years before Christ, back to the remotest Epoch.*

From any multiple of 532, preferring the multiple next greater than the given Year, Subtract the given Year B. C. less 1, (but the year itself, if denoted astronomically.) The Difference will be a Julian Year *after* Christ holding a *corresponding* place with it in the *Dionysian Period*, (or product of the solar cycle of 28, by the lunar cycle of 19 years,) Then the foregoing Rules applied to the Year *after Christ*, thus found, must yield both the Days of the Week and the Epact, correctly, for the given Year *before* Christ. If the year *after* Christ be a multiple of 4, the year *before* Christ, (though an odd number) is also a leap year.

EXAMPLES.

What day of the week was May the 28th in the 585th Year B. C., Old Style?

532 × 2 = 1064
585 — 1 = 584

Difference	480	or like year *after* Christ.
One fourth part,	120	
Eq. in column A	5	
No. for May, Table B	1	
Day of Month	28	
Divide by 7)	634	
Remainder	4	or Wednesday.

(See Article "Cycle," in Rees's Cyclopedia.) It is a *Leap* Year, and its Epact will be found, by the Tablet, to be 25.

What day of the week was January the 1st, in the Year 6857, B. C.? (reckoned astronomically.)

532 × 13 = 6916
6857

Difference	59	or like year *after* Christ.
One fourth part	14	
Eq. in column A	5	
No. for Jan., Table B	0	
Day of the Month	1	
Divide by 7)	79	
Remainder	2	or Monday.

See Delambre, who says "*that* year *begins* and *ends* on Monday." It is, of course, *common*, and its Epact will be found to be 22.

2*d.* *A Rule for Years of the Julian Period, or (A. J. P.,) both before and after Christ.*

$$\left.\begin{array}{l}\text{4714 less the Chronological Year B. C.}\\ \text{4713 less the Astronomical Year B. C.}\\ \text{4713 added to the Year of Christ A. D.}\end{array}\right\} = \text{the Year of the Julian Period (or A. J. P.)}$$

$$\left.\begin{array}{l}\text{To A. J. P. add 19, or from A. J. P. deduct 9}\\ \text{To A. J. P. add 18, or from A. J. P. deduct 1}\end{array}\right\} \text{and to the } \textit{Sum} \text{ or } \textit{Difference}\text{, as if it were a Julian A. D.,}$$

$$\text{Apply} \left\{\begin{array}{l}\text{the Civil Rule}\\ \text{the Church Rule}\end{array}\right\} \text{of the Tablet for finding} \left\{\begin{array}{l}\text{the Day of the Week, or Feria.}\\ \text{the age of the Moon, or Epact.}\end{array}\right\}$$

The Sum, or Difference *lessened* by any multiple of 532 (such as 3724) yields the same answers.

EXAMPLES.

A. J. P. 3938	July 1st,	year B. C.	776	Chronological Epoch of the Olympiads,	Feria	2	or Monday,	Epact 14.
3961	April 21,	"	753	" " Rome	"	2	Monday,	" 28
3967	Feb. 26,	"	747	" " Nabonassar	"	4	Wednes'y,	" 4
4104	Sept. 30,	"	610	Eclipse predicted by Thales (F. Baily)	"	6	Friday,	" 18
4129	May 28,	"	585	Eclipse Art. "Cycle," Rees's Cyclop.	"	4	Wednes'y,	" 25
4669	Jan. 1,	"	45	Reform. Cal. of Jul. Cæsar (bissextile)	"	6	Friday,	" 23
4713	Jan. 1,	"	1,	or Year 0 of Astronomers (bissextile)	"	5	Thursday,	" 29
5335	July 16,	A. D.	622	Epoch of the Hegira	"	6	Friday,	" 4

$$\text{Ex. first, A. J. P. 3938} + \left\{\begin{array}{l}19 = 3957 \text{ A. D.}\\ 18 = 3956 \text{ A. D.}\end{array}\right\} \text{And} \left\{\begin{array}{l}3957\\ 3956\end{array}\right\} - 3724 \text{ (or } 532 \times 7) = \left\{\begin{array}{l}233 \text{ A. D.}\\ 232 \text{ A. D.}\end{array}\right.$$

$$\text{A. J. P. 3938} - \left\{\begin{array}{l}9 = 3929 \text{ A. D.}\\ 1 = 3937 \text{ A. D.}\end{array}\right\} \text{And} \left\{\begin{array}{l}3929\\ 3937\end{array}\right\} - 3724 \text{ (or } 532 \times 7) = \left\{\begin{array}{l}205 \text{ A. D.}\\ 213 \text{ A. D.}\end{array}\right.$$

Results as above.

3d Rule. As a means of *proving* all the foregoing operations, or *in lieu* of them, with a view, chiefly, to save figures in computing the Epact, EMPLOY the following formulæ, which do not refer either to the cycle of the sun, or to that of the Golden Numbers, now in common use, beginning with the year $1+9$, and the year $1+1$, respectively, but to a succession of cycles of 28 and 19, in which the years corresponding with the *first* year of the Christian Era must always be numbered the *first* of each cycle also.

FORMULÆ.

Call any astronomical year *before Christ*, y and any A. D., or year *after Christ*, Y } and their Remainder after division by either cycle, r

If $r = 0$, keep it so; except in Old Style Years, when, if 19 be the Divisor, change 0 to 19.

Then $28 - \left(\frac{y}{28}\right)_r$, and $19 - \left(\frac{y}{19}\right)_r$, will express y's No. in each cycle in the Era B. C.

And $\left(\frac{Y}{28}\right)_r$, and $\left(\frac{Y}{19}\right)_r$, will express Y's No. in each cycle in the Era *after Christ*.

Apply the RULES of the Tablet, on each side, to the Cyclic No. thus found, as if it were the given year, using always the secular equations A and C, belonging to the given ERA, and the results will be uniformly the same, as in the examples heretofore stated.

When the Cyclic No. on the Civil side is a multiple of 4, the year is leap, unless it be a New Style 100th year unmarked with an asterisk.

When the Cyclic No. on the Church side is 19, "1 less than the 19th part" becomes 0, and the Exception to the Rule for finding the Epact is thus eliminated.

EXAMPLES.

1. *Required both the Old and New Style Easter of A. D.* 1848, (*being a multiple of* 28.)

Julian Year.	Easter.	Gregorian Year.	Easter.
28) 1848	19) 1848	28) 1848	19) 1848
Remainder 0	Remainder 5	Remainder 0	Remainder 5
its 4th 0	5 × 10 = 50	its 4th 0	5 × 10 = 50
A 5	C 0	A 0	C 0
Month 6	30) 55	Month 6	30) 55
Day 10	Epact 25	Day 18	Epact 25
7) 21 Rem.	From 35	7) 24 Rem.	From 43
Saturday, 0 or 7	Term April 10	Tuesday 3	Term April 18
From 8		From 8	
1	to Sunday 1	5	to Sunday 5
	Answer, April 11		Answer, April 23

2. *Required both the Old and New Style Easter of A. D.* 2698, (*being a multiple of* 19.)

Julian Year.	Easter.	Gregorian Year.	Easter.
28) 2698	19) 2698	28) 2698	19) 2698
Remainder 10	Remainder 0 =	Remainder 10	Remainder 0 (kept 0)
its 4th 2	Divisor 19	its 4th 2	0 × 10 = 0
A 5	19×10=190	A 1	C 26
Month 6	C 0		
		Month 6	30) 26
Day 5	30) 209	Day 17	
7) 28 Rem.	Epact 29	7) 36 Rem.	Epact 26
Saturday 0 or 7	From 34	Sunday 1	From 43
From 8	Term April 5	From 8	Term April 17
1	to Sunday, 1	7	to Sunday 7
	Answer, April 6		Answer, April 24

In the first example, the difference between the styles is 12 days, and in the second, it is 18 days. It so happens, therefore, that, in both these cases, the Julian and Gregorian Easters will be celebrated on the very same day. The year 2698 is the last, according to Mr. De Morgan, in which such a coincidence will occur. See note at the foot of page 19, in his Essay on the Ecclesiastical Calendar. See also page 105 of this Memoir.

The Rule for the Gregorian Epact, it will be perceived, becomes universally the following, *To* 11 *times the Cyclic No. add the Equation in Column C; and reject* 30*s from the Sum.*

The Rule for the Feria, (or day of the week,) though not attended with an equal economy of Figures, suggests the practical convenience of marking for remembrance, in any current century, those years which terminate cycles of 28, (such as 1820, 1848, and 1876, in the present century,) in which case the Cyclic No. for any intermediate year, may be promptly known, and the day of the week be thence deduced by an easy *mental* process. For instance, the Cyclic No. for 1847 is 27, (or 27 years beyond 1820,) and the other figures to be added to it are so few and small, that ordinary questions may be solved by the Rule without putting pen to paper, more especially in the present century, whose secular equations are, on both sides of the Tablet, null until 1900.

A similar expedient might be adopted with the years 1805, 1824, 1843, 1862, 1881, each ending cycles of 19, and the Epacts for intermediate years be *mentally* computed with like facility. Thus the Cyclic No. for 1847 is 4, (or four years beyond 1843,) and the Epact is 14, or 4 times 11 *lessened* by 30.

Without departing, however, from the original form of the Tablet, the work may be somewhat abbreviated by noting those years only which close at once, centuries and cycles, (such as 1400,1800, &c., on one side, and 1900, 3800, &c., on the other,) and by using, in computation, the years beyond those epochs respectively. Thus 447, 448, &c., yield the same feriæ as A. D. 1847, 1848, &c.; and, in the coming century, 1, 2, 3, &c., will yield the same Epacts as A. D. 1901, 1902, &c.

4th. A Rule for the Solution, by the foregoing Tablet, of Converse Questions, viz.,

To find on what day of the *Month* a given Day of the *Week* first falls in any *Month* in any year.
Omit the day of the Month in the fifth line, and divide the Sum of the four other lines by 7; Subtract the Remainder from the numerical *Day of the Week*, increased, if needful, by 7, The Difference will be the ANSWER. But in January and February of Leap Year take the Remainder from the *succeeding* day of the Week, increased, if needful, in like manner.

EXAMPLES.

What Day of the Month was the first Monday in Dec. 1846, *the day of the meeting of Congress?*

The Remainder, by the above Rule, will be . . . 2
Monday 2 — 2 = 0: Increase, therefore, the day by 7.
Then 9 — 2 = 7. Answer, December the 7th.
Proof—The Tablet shows that the month began on TU.

What Day of the Month was the first Thursday in Feb., 1844? (*being a Leap Year.*)

The Remainder by the above Rule, will be 5
5 taken from 6, (Friday) leaves for the answer 1
or the *first* day of that Month.
See De Morgan's Essay, page 16, Example 2d.

NOTE.—Any reference, in Leap Years, either to *preceding* or *succeeding* Days of the Week in January and February may be avoided by substituting the Monthly No. of July for January, and that of August for February, each No. being half a year distant from that which it takes the place of.

In the last example, for instance, had the August No. 2 been used instead of the February No. 3, the Remainder would have been 4, which taken from Thursday, or 5, would have given the same result.

APPENDIX TO MR. McILVAINE'S MEMOIR.

Read July 16th, 1847.

I beg the Society's acceptance of the accompanying Cards, containing a new plan of a Perpetual *Civil* Calendar, &c., in which, still dispensing with Dominical Letters, and substituting for them Yearly Numbers, (always their *complement* to 8, if we read 0 as equal to 7,) the same results may be obtained by mere *inspection*, as those requiring computation according to the scheme heretofore presented by me. The equivalence of the two methods will be readily recognised by the following comparison of them, which is universally applicable.

The Yearly Number, by the former plan, would be the Remainder on division by 7 of the Sum of the Year, its fourth part (omitting fractions,) and the secular Equation in Column A; which Remainder, for the New Style year 1847, after Christ, is 5, the same as the Yearly Number here standing in Table C, at the intersection of the *line* of A with the *column* of B. The same line (be it observed) answers for a whole century.

Now for the names of the months of Table E in this plan, substitute the Monthly Numbers 0, 1, 2, 3, 4, 5, 6, of Table B in the former, and the two processes become virtually identical, thus:

The 18th of June, 1847, found by the former, would be $\left(\frac{5+4+18}{7}\right)_r = \left(\frac{27}{7}\right)_r = 6$ or Friday. By the present, counting 5 onwards from 18 in Table C, we reach a column containing 23; then descending the *column* to Table E, we find in the *line* of June the Day of the Week to be Friday, corresponding with the Remainder 6. If, in the blank space between the Examples and Table E, and in line with the respective months, their seven monthly numbers were so arranged as to form a short column, it would, perhaps, be more clearly seen how and why, the relative position of the several Tables of page 129 effects the same object as the Rule on the Civil side of the Perpetual Calendar, at page 106.

I have given no examples of *New Style* years, either before the Christian Era, or between it and the year 1582, because that mode of reckoning, though well calculated to reveal, and to measure approximately, the chief defect of the Julian Calendar, is not customary in chronology, and, being somewhat speculative, might perplex, rather than assist an inquirer; but, in Old Style years before Christ, and in New Style years until the Gregorian reckoning shall be modified, the wide range of these five Tables may be very satisfactorily shown by two of the examples which have been already worked in a different manner, namely the 1st of January, in the astronomical year 6857, B. C., at page 125, and the 16th of April, A. D 50000, at page 123. Both their centurial figures lie beyond the limits of Table A; but 68, yielding, on division by 7, the Remainder 5, belongs to the same line with 5, O.S. B.C.; and 500, yielding, on division by 4, the Remainder 0, belongs to the same line with 16, N.S. A.C. The Yearly Number in the first case, (using, agreeably to the 2d Exception, 43 instead of 57, in Table B,) is 1, and the answer is Monday. The Yearly Number, in the second case, is 0, and the answer is Sunday.

This device is accordingly verified by the formulæ of Delambre to the same extent as the one from which it has been drawn; and I solicit permission to occupy, with a copy of it, an additional page of the Transactions.

NEW PLAN OF A PERPETUAL CIVIL CALENDAR, JULIAN AND GREGORIAN,

SHOWING, BY INSPECTION,

THE AGREEMENT OF *MONTHLY DATES* WITH *DAYS OF THE WEEK* IN ANY *YEAR* BEFORE OR AFTER THE CHRISTIAN ERA.

USE OF THE TABLES A, B, C, D, E.

Given the *monthly date* to find *the day of the week.*

1. In A look for the *horizontal line* containing the CENTURIAL FIGURES, and In B for the *vertical column* containing the remaining PART OF A CENTURY, In C, where line and col. *meet*, is the YEARLY NUMBER, which keep in mind.
2. In D look for the DAY OF THE MONTH, from which *day*, (but exclusive of it,) count onwards *as many days as the* YEARLY NUMBER, you have just noted. Then going down the *column* you have reached to the *line* of the MONTH in E, you will find, where they *meet*, the DAY OF THE WEEK sought.

EXCEPTIONS.

1. In Leap Years *use* the line of { JULY for JAN. (the bottom line for the top,) AUG. for FEB. (the line next above FEB.)
2. In the Era B. C. *use* the *complement* to 100 of the *given* PART OF A CENTURY, and, consequently, for centesimal years, *use* the column marked * 00, at the foot of B, instead of the column marked *0 at the head of that Table.

NOTE.—ASTRONOMICAL years B. C., are always 1 *less* than CHRONOLOGICAL, and must be so denoted. Thus the chronological years 1, 2, 3, &c. are the same as the astronomical years . . . 0, 1, 2, &c.

A

WHOLE CENTURIAL FIGURES.

BEFORE CHRIST. ASTRONOMICAL. *New Style.*	BEFORE CHRIST. ASTRONOMICAL. *Old Style.*	AFTER CHRIST. *Old Style.*	AFTER CHRIST. *New Style.*
10 6 2	20 13 6	0 7 14	17 21 25 29 33 37
	19 12 5	1 8 15	
9 5 1	18 11 4	2 9 16	18 22 26 30 34 38
	17 10 3	3 10 17	
*8 *4 *0	16 9 2	4 11 18	15 19 23 27 31 35 39
7 3	15 8 1	5 12 19	*16 *20 *24 *28 *32 *36 *40
	14 7 0	6 13 20	

Any other CENTURIAL FIGURES yielding, on division by 7, in Old Style, and by 4, in New Style, the same *Remainder* as those above do, when so divided, belong to the *same line of the same ERA.*

B

PARTS OF A CENTURY. Leap years marked thus *, *but* N. S. 100th years are *not* leap unless the CENT. FIGS. have a *.

				* 0	1	2
3		* 4	5	6	7	
* 8	9	10	11		* 12	13
14	15		* 16	17	18	19
	* 20	21	22	23		* 24
25	26	27		* 28	29	30
31		* 32	33	34	35	
* 36	37	38	39		* 40	41
42	43		* 44	45	46	47
	* 48	49	50	51		* 52
53	54	55		* 56	57	58
59		* 60	61	62	63	
* 64	65	66	67		* 68	69
70	71		* 72	73	74	75
	* 76	77	78	79		* 80
81	82	83		* 84	85	86
87		* 88	89	90	91	
* 92	93	94	95		* 96	97
98	99		* 00			

C

YEARLY NUMBERS.

I.	II.	III.	IV.	V.	VI.	O.
O.	I.	II.	III.	IV.	V.	VI.
VI.	O.	I.	II.	III.	IV.	V.
V.	VI.	O.	I.	II.	III.	IV.
IV.	V.	VI.	O.	I.	II.	III.
III.	IV.	V.	VI.	O.	I.	II.
II.	III.	IV.	V.	VI.	O.	I.

CONVERSE USE OF THE TABLES.

Given *the day of the week* to find the *monthly dates.*

1. In A, B, and C obtain the YEARLY NUMBER, as you were before directed.
2. In E look for the *line* containing both THE NAME OF THE MONTH and THE DAY OF THE WEEK, from which *day*, (excluding itself,) count backwards *in that line as many days as the* YEARLY NUMBER you have noted. Then going up the *column* you have reached to D, you will find all the *corresponding dates of that* MONTH (four or five,) presented vertically, at one view. But attend to both the EXCEPTIONS as above stated.

D

DAYS OF THE MONTH.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	highest Sum of C and D.				

EXAMPLES.

	A B	*Comp. to* 100.	C	D	Sum of C & D	*Except first*	Ans. E	EPOCHS.
O. S. B. C.	775	25	II.	1 July	3		M.	Olympiads
	752	48	III.	21 Apr.	24		M.	Rome founded.
	746	54	III.	26 Feb.	29		W.	Babylon founded.
	44	*56	VI.	1 Jan.	7	*July	F.	Reformed Cal. of Julius Cæsar.
	0	*00	V.	1 Jan.	6	*July	TH.	Astron. year 0 = Chronolog. 1.
O. S. A. C.	1		VI.	1 Jan.	7		SA.	CHRISTIAN ERA begins.
	622		V.	16 July	21		F.	Hegira.
	1582		I.	5 Oct.	6		F.	Oct. 5th made 15th, N. S. at Rome.
	1732	*	O.	11 Feb.	11	*Aug.	F.	} WASHINGTON born.
N. S. A. C.	1732	*	III.	22 Feb.	25	*Aug.	F.	}
	1752		O.	14 Sep.	14		TH.	Sep. 3 O. S. made 14th in G. Brit.
	1776		II.	4 July	6		TH.	Independence declared by U.S.A.
	1847		V.	6 Dec.	11		M.	Congress meets first M. in Dec.
	1800		III.	1 Jan.	4	com. yr	W.	Centurial figs. 18 have *no* star.

E

MONTH OF	DAYS OF THE WEEK.						
JAN. OCT.	SU.	M.	TU.	W.	TH.	F.	SA.
MAY	M.	TU.	W.	TH.	F.	SA.	SU.
AUG.	TU.	W.	TH.	F.	SA.	SU.	M.
FEB. MAR. NOV.	W.	TH.	F.	SA.	SU.	M.	TU.
JUNE	TH.	F.	SA.	SU.	M.	TU.	W.
SEP. DEC.	F.	SA.	SU.	M.	TU.	W.	TH.
APR. JULY	SA.	SU.	M.	TU.	W.	TH.	F.